On Optimal Feedback Allocation in Multichannel Wireless Downlinks

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ABSTRACT

This paper studies feedback resource allocation in the downlink of a Frequency Division Duplex (FDD) multichannel wireless system. We consider a downlink network with a single base station, L shared channels and N mobile users. Throughput optimal algorithms like the MaxWeight scheduling in general require the complete channel state information (with $N \times L$ link states) for scheduling, which could be unaffordably expensive when the number of users is large. In this paper, we consider the scenario where the base station allocates only limited uplink resource for acquiring channel state information. We first show that to support a fraction $(1 - \epsilon)$ of the full throughput region (the throughput region with full channel state information), the base station needs to acquire at least $\Theta(L)$ link states at each time slot. We then propose a *Weight Based Feedback allocation*, named WBF, and show that WBF together with the MaxWeight scheduling achieves a fraction $(1 - \epsilon)$ of the full throughput region by acquiring at most $\Theta\left(L \log \frac{1}{\epsilon}\right)$ link states per time slot.

Categories and Subject Descriptors

C.2.1 [**Computer-Communication Networks**]: [Network Architecture and Design – *Wireless Communication*]

General Terms

Algorithms, Performance

Keywords

Limited feedback, downlink scheduling, throughput optimality

1. INTRODUCTION

This paper considers wireless downlinks using multi-carrier techniques, e.g., Orthogonal Frequency Division Multiplexing (OFDM). Multi-carrier techniques divide wireless resource into parallel channels, each channel corresponding to a spectrum block. To exploit multiuser diversity in multichannel downlink networks, the base station needs to acquire the channel state information (CSI) from mobiles for throughput-optimal scheduling [1, 5]. The mobiles' CSI is usually unknown at the base station, especially in popular Frequency Division Duplex (FDD) systems which lack of channel reciprocity. A common method to acquire the downlink CSI is to allocate a part of uplink resource to mobiles to report their CSI. However it becomes increasingly unaffordable as we explain in the following paragraph.

Consider a multichannel downlink network with 10MHz uplink and downlink bandwidths respectively, divided into 50 channels (or called resource blocks), and 50 mobiles. This network may have a uplink peak rate of 48Kbits per subframe (1 ms) and a downlink peak rate of 80Kbits per subframe [4]. In this paper, we distinguish the concepts of "channel" and "link". A "channel" is referred to a certain *frequency band* and a "link" is referred to the *wireless connection* between a mobile and the base station over a specific frequency band. Therefore, the downlink network we introduced has 50 channels and 2,500 links. Assume each link state is represented by 4 bits data. Then the full CSI requires $2,500 \times 4 = 10K$ bits per subframe, which is more than 20% of the uplink capacity and 12.5% of the downlink capacity.[1] We note that for slow-fading scenarios (e.g., indoor fixed-location transmissions) where channel coherence time is large, channel states can be reported at a slower time scale (at the time scale of channel coherence time). But nevertheless, the amount of feedback resource required by the full CSI feedback still linearly increases in term of the number of channels as well as the number of mobiles, which can results in a significant communication overhead. Since a 10MHz spectrum may cost billions of dollars (Verizon paid 4.74 billion for a block of 22MHz bandwidth in 2008), full CSI feedback is clearly unaffordable.

In current and next generation cellular standards, such as 802.16e [2], 802.16m [3] and 3GPP LTE [4], the system only allocates limited bandwidth for CSI feedback and mobile users need to share the limited bandwidth. In this paper, we consider a multichannel downlink network with L channels and N mobiles. The system is operated in a slotted fashion such that the base station makes feedback/scheduling decisions once every time slot. The duration of a time slot can be one subframe or multiple subframes (e.g., chan-

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[1]The control information like CSI feedback is usually transmitted at a reliable base rate, which costs more bandwidth than a regular data transmission for the same amount of information bits.

nel coherence time). We define the feedback resource F to be the number of link states that are allowed to be reported to the base station during each time slot. For example, the full feedback information requires $F = NL$.

We denote the network throughput region by Λ_{full}, which is the set of supportable traffic loads[2] with full CSI. In this paper, we address the following fundamental questions:

1. *What is the minimum amount of feedback resource required to support a fraction $(1-\epsilon)$ of the full throughput region?*

2. *What is the optimal way to allocate the limited feedback resource among the mobile users?*

Note that the first question deals with the fundamental limit of a multichannel downlink network, and the second question deals with the design of an efficient feedback algorithm.

1.1 Main Contributions

The main contributions of this paper include:

1. We first establish an algorithm-independent lower bound on the amount of feedback resource required to achieve a fraction $1-\epsilon$ of the throughput region. We show that to achieve a fraction $1 - \epsilon$ of the full network throughput region, the base station needs to acquire at least $\Theta(L)$ link states per time slot.

2. We then develop a weight based feedback allocation algorithm named WBF, where the base station allocates feedback resource at the beginning of each time slot according to the queue-lengths at the base station and the channel statistics. We prove that WBF combined with the MaxWeight scheduling achieves a $1 - \epsilon$ fraction of the full throughput region with $F = \Theta\left(L \log \frac{1}{\epsilon}\right)$ per time slot. We comment that the feedback resource required under WBF is a function of L and ϵ, and is independent of the number of mobiles (N) in the network. The dowlink/uplink capacities of an L channel system scales linearly as L, so WBF requires only a constant communication overhead. The full CSI feedback on the other hand is not scalable because it needs to acquire NL feedback resource per time slot.

We further comment that WBF is an interesting combination of a centralized resource allocation and opportunistic feedback report. Under WBF, at the beginning of each time slot, the base station allocates the feedback resource to mobiles (i.e., determining the number of link states a mobile can report), which is a centralized allocation, and then the mobiles select their best links to report, which is an opportunistic feedback report.

1.2 Related Work

There has been a lot of interests in developing joint feedback and scheduling algorithms for downlink wireless systems. Opportunistic feedback has been proposed in [8,10,11, 18], where all mobiles share a common feedback channel and contend for the feedback channel if the channel state exceeds a pre-defined threshold. Opportunistic feedback algorithms

are designed primarily for exploiting multiuser diversity and usually assume that all users are infinitely backlogged.

Another approach is to let the base station allocate the feedback resource. For single channel networks, joint channel-probing and scheduling algorithms have been proposed and analyzed in [6,9,12–17,19]. In [9,12–14,16,17,19], the focus is to consider joint channel probing and scheduling problem for a time-division duplex (TDD) system. In such a system, the downlink data transmission and CSI feedback share the same radio frequency. After sending the probing request, the base station needs to wait for the feedback over the same reverse channel before transmitting downlink data. The authors in [12] modeled the joint probing and scheduling as a minimum cost problem and developed an algorithm with polynomial complexity. In [14], structural properties of optimal probing policy have been characterized, and in [17], a sequential probing with one-bit per user is studied for an OFDMA downlink system. In [19], the authors developed a throughput optimal algorithm with limited feedback. In [6, 15], the authors studied the joint CSI feedback and scheduling problem for Frequency Division Duplex (FDD) systems, in which CSI feedback could be transmitted simultaneously with downlink data transmissions. The most related work is [15], where the optimal feedback-scheduling scheme for a single-channel downlink is derived. This paper distinguishes itself from previous works by studying joint CSI feedback and scheduling for a multi-channel, multi-rate downlink network. The readers may ask whether the feedback-scheduling algorithms for single-channel networks can be directly applied to a multichannel network by treating the multichannel network as L single channel networks. This approach however does not exploit the fact that a mobile can measure the states of all L links after a pilot signal is broadcast from the base station, and can decide which links to report (e.g., the mobile can report its best links). This additional degree of freedom makes the design of feedback-scheduling algorithms in multichannel networks fundamental different from that in single-channel networks. For example, consider a multichannel network with symmetric channels and mobile users. The algorithms in [19,15] will probe the same subset of mobiles on each channel, so a mobile reports either all its link states or none. As we will see later in this paper, such schemes are sub-optimal because with a high probability, only some links of a mobile are in good states. An optimal feedback scheme should allocate only limited feedback resource to a mobile, an amount sufficient for the mobile to report all good link states.

2. SYSTEM MODEL AND NOTATIONS

We consider the downlink of a FDD cellular network with one base station, L channels and N mobile users. Each user is associated with a downlink data flow. A separate queue is maintained for each flow at the base station. Time is slotted. We use $Q_i(t)$ to denote the length of the queue for mobile user i at the beginning of time slot t. The N flows are served by the L shared channels. This wireless downlink system can be modeled as a discrete-time queueing system with N queues and L servers as shown in Figure 1.

We use (ij) to denote the link connecting the base station and mobile i using channel j. The following notations are adopted throughout this paper. Denote $X_{ij}(t)$ as the link state of channel j at user i at time slot t. In practice, the link state is in the form of the maximum supportable data

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[2]A traffic load is said to be supportable if there exists a scheduling algorithm under which the amount of backlogs at the base station is bounded.
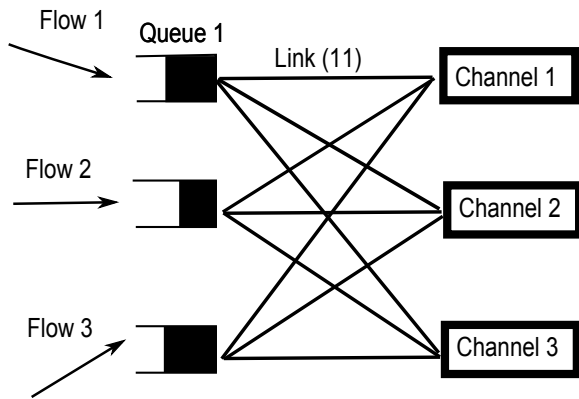
Figure 1: A three-channel, three-user queuing system

rate over that link at time slot t. In practical systems, there are a finite number of modulation and coding schemes, so we assume each link has R possible link states with rates $\{r_1, r_2, r_3, ..., r_R\}$, where r_k is the service rate if the link is in state k. For simplicity, we assume r_k is sorted in a descending order such that $r_k < r_l$ if $k > l$. We assume the link states are independently distributed across users, and independently and identically distributed across channels for the same user. For a mobile user i, we denote that the probability that link (ij) is in state k is

$$\Pr\left(X_{ij}(t) = r_k\right) = p_{ik}.$$

Note that for a given user, this probability is assumed to be independent of j because we assume that link states are independent and identically distributed across channels for each user. We further assume that the link state distributions are known at the base station[3]. We also assume that $r_R > 0$, $p_{ik} \leq p^{\max} \leq 1$ for all i and k, $p_{ik} \geq p^{\min} > 0$ for all i and k,[4] and $\sum_k p_{ik} \leq 1$ where $\sum_k p_{ik} < 1$ is allowed so that a link can be off with a certain probability.

The system allocates limited uplink resource for link state feedback. In each time slot, at most F link states can be reported to the base station. We assume mobile i knows all link states $X_{ij}(t)$[5] and can choose a subset of them to report to the base station.

We denote $Y_{ij}(t)$ to be the feedback decision of mobile i on link state $X_{ij}(t)$, i.e.,

$$Y_{ij}(t) = \begin{cases} 1, & \text{if } X_{ij}(t) \text{ is reported by user } i \text{ ;} \\ 0, & \text{otherwise.} \end{cases}$$

We further denote $Z_{ij}(t)$ to be the scheduling decision of the

base station on link (ij), i.e.,

$$Z_{ij}(t) = \begin{cases} 1, & \text{if channel } j \text{ is allocated to user } i \text{ at slot } t; \\ 0, & \text{otherwise.} \end{cases}$$

We assume one time slot is the finest granularity for feedback and scheduling, i.e., at one time slot, a channel can be allocated to one and at most one user. As a result,

$$\sum_{i=1}^{N} Z_{ij}(t) \leq 1, \forall j, t.$$

We further assume a transmission to user i over channel j could not be fulfilled unless the link state is reported. This assumption however can be relaxed, and we will discuss the extension in Section 6. Using $D_i(t)$ to denote the service rate allocated to user i at time slot t, we have

$$D_i(t) = \sum_{j=1}^{L} X_{ij}(t)Y_{ij}(t)Z_{ij}(t).$$

Next we define $A_i(t)$ to be the number of packets arriving at time slot t for mobile user i. We assume $A_i(t)$s are stationary and bounded random variables, which are independent across users and time slots and independent of link states $X_{ij}(t)$s, and $a_i = \mathbf{E}[A_i(t)]$. We further assume packets arrive at the base station at the beginning of a time slot and are served at the end of the time slot.

The evolution of queue length Q_i can be written as

$$Q_i(t + 1) = (Q_i(t) + A_i(t) - D_i(t))^+$$

where $(x)^+ = \max\{x, 0\}$. We finally recall that Λ_{full} is the network throughput region such that given any $(a_1, \ldots, a_N) \in \Lambda_{full}$, the network can be stabilized under some scheduling algorithm with full CSI. We summarize the notations in the appendix.

3. ALGORITHM-INDEPENDENT LOWER BOUND

In this section, we study the fundamental impact of limited feedback resource on network throughput for any $1 > \zeta > 0$. We are interested in knowing *how many link states the base station needs to acquire to support $1 - \epsilon$ fraction of the full throughput region?* The theorem below answers this question by providing an algorithm independent lower bound on F.

THEOREM 1. *To support $(1-\epsilon)$ fraction of the full throughput region, the base station needs to acquire at least*

$$(1 - \epsilon)\left(1 - (1 - p^{\min})^N\right)L$$

link states per time slot.

PROOF. Assuming that for each flow i, packets arrive with a constant rate a_i. We first show that traffic load

$$a_i = (1 - \zeta)\frac{\left(1 - (1 - p^{\min})^N\right)r_1 L}{N} \text{ for all } i$$

is always in the network throughput region. To prove this, we consider the following scheduling algorithm:

- At time t, the base station constructs a set $\Psi_j(t)$ for each channel j such that if $X_{ij}(t) = r_1$, then user i is selected into $\Psi_j(t)$ with probability $p^{\min}/p_{i1}$. We

construct $\Psi_j(t)$ such that each mobile has the same probability $p^{\min}$ to be selected into the set, and the selected mobiles have $X_{ij}(t) = r_1$.

- If $\Psi_j(t)$ is not empty, then the base station randomly and uniformly selects a mobile user i from $\Psi_j(t)$ and serves the user with rate r_1.

Under the scheduling algorithm above, the probability a user i is selected into set $\Psi_j(t)$ is

$$p_{i1} \times \frac{p^{\min}}{p_{i1}} = p^{\min},$$

which is identical across the mobiles. The probability set $\Psi_j(t)$ is not empty is

$$\left(1 - (1 - p^{\min})^N\right).$$

At each time slot, at most one user can be served over channel j, and the service rate is r_1. Therefore, the average service rate (over channel j) allocated to user i is

$$\frac{\left(1 - (1 - p^{\min})^N\right) r_1}{N}.$$

Since the network consists of L channels, the average service rate a user receives is

$$\frac{\left(1 - (1 - p^{\min})^N\right) r_1 L}{N}.$$

Hence, traffic load

$$a_i = (1 - \zeta)\frac{\left(1 - (1 - p^{\min})^N\right) r_1 L}{N} \text{ for all } i$$

can be supported by the scheduling algorithm above, and the traffic load lies in the network throughput region.

Now under the limited feedback scheme, a link is scheduled only if the link state is reported to the base station. Therefore, at most F links are scheduled at one time, and the maximum link rate is r_1, which implies that the maximum sum throughput we can support is Fr_1. We then can obtain that if F is sufficient for supporting a fraction $1 - \epsilon$ of the full throughput region, then:

$$
\begin{aligned}
Fr_1 &\geq (1 - \epsilon)N(1 - \zeta)\frac{\left(1 - (1 - p^{\min})^N\right) r_1 L}{N} \\
&\geq (1 - \epsilon)(1 - \zeta)\left(1 - (1 - p^{\min})^N\right) Lr_1 \\
&\geq (1 - \epsilon)\left(1 - (1 - p^{\min})^N\right) Lr_1,
\end{aligned}
$$

where the last inequality holds because the second inequality holds for any $\zeta > 0$. Thus, we can conclude that to achieve a fraction $(1 - \epsilon)$ of the full throughput region, the feedback resource F should satisfy

$$F \geq (1 - \epsilon)\left(1 - (1 - p^{\min})^N\right) L. \tag{1}$$

□

4. ORDER OPTIMAL FEEDBACK ALLOCATION ALGORITHM

Theorem 1 implies that to achieve a near-optimal throughput region, an amount of $\Theta(L)$ feedback resource is necessary. The next question therefore is *does there exist an algorithm that can support a fraction $1 - \epsilon$ throughput region*

with $\Theta(L)$ link feedbacks? The answer to this question is *Yes!* We propose the following weight based feedback allocation algorithm, and prove that combined with the MaxWeight scheduling, the algorithm supports $1 - \epsilon$ of the full throughput region with $F = \Theta(L \log 1/\epsilon)$, which is much smaller than the full feedback requirement $F = NL$.

Let $m_i(t)$ denote the number of link states mobile user i can report at time t, so $\sum_i m_i(t) \leq F$. We note there are two challenges in deciding $m_i(t)$:

(i) The feedback allocation is done by the base station, who does not know $X_{ij}(t)$. So mobile i may not have $m_i(t)$ "good" channels to report given $m_i(t)$ feedback resource; and

(ii) The mobile users cannot cooperate with each other to choose $Y_{ij}(t)$ because they only know their own link states. So for some channel, the base station may receive multiple feedbacks from different mobiles, and for some other channels, the base station may receive no feedback.

Because of the two issues above, designing the feedback resource allocation algorithm becomes interesting and challenging.

Weight Based Feedback (WBF):

- At the beginning of time slot t, the base-station sorts

$$\gamma_{ik}(t) \triangleq Q_i(t)r_k$$

in a descending order. If there is a tie, the preference is given to small i. Define i_n and k_n to be the i and k associated with the γ at the n-th position. So $\gamma_{i_n k_n}(t) > \gamma_{i_{n+1} k_{n+1}}(t)$, or $\gamma_{i_n k_n}(t) = \gamma_{i_{n+1} k_{n+1}}(t)$ for $i_n < i_{n+1}$.

The base station then allocates the feedback resource to mobile users iteratively:

- Step 0: The base station sets $\sigma = 1$, $m_i^0 = 0$ for all i, and $F^0 = F$.

- Step 1: The base station sets

$$m_i^\sigma = \begin{cases} \min\{F^{\sigma-1}, (1+\delta)p_{i_\sigma, k_\sigma}L, L - m_i^{\sigma-1}\} \\ \quad + m_i^{\sigma-1}, \quad \text{if } i = i_\sigma \\ m_i^{\sigma-1}, \quad \text{otherwise.} \end{cases}$$

and

$$F^\sigma = F^{\sigma-1} - (m_{i_\sigma}^\sigma - m_{i_\sigma}^{\sigma-1}).$$

In other words, the base station increases the amount of feedback resource allocated to mobile i_σ by $(1 + \delta)p_{i_\sigma, k_\sigma}L$ if there are sufficient remaining feedback resource.

- Step 2: If $F^\sigma \neq 0$, increase σ by one and go to step 1. Otherwise, the base station finalizes the feedback resource allocation such that

$$m_i(t) = m_i^\sigma$$

for all i.

- The base station communicates the decision $m_i(t)$ to mobile user i and allocates corresponding uplink resource to allow mobile i to report $m_i(t)$ link states.

- Mobile i, after receiving the feedback resource allocation decision $m_i(t)$, reports the best $m_i(t)$ link states to the base station.

$\diamond$

Example: The feedback resource is allocated according to the value of $\gamma_{ik}(t)$. Consider a simple example with two mobile users and two channels. We assume that $p_{ik} = 0.5$ for all i and k so that $p_{ik}L = 1$ for all i and k, which is the average number of links in state k for user i. So on average, user 1 has one link with rate $r_1 = 2$ and one link with rate $r_2 = 1$. We also assume $\delta = 0$ for simplicity. Then the feedback resource is allocated according to Table 1. Larger γ_{ik} has the preference to be selected; and when γ_{ik} is selected, mobile i can report $p_{ik}L$ link states.

	$r_1 = 2$	$r_2 = 1$
$Q_1(t) = 10$	$\gamma_{11} = 20, p_{11}L = 1$	$\gamma_{12} = 10, p_{12}L = 1$
$Q_2(t) = 8$	$\gamma_{21} = 16, p_{21}L = 1$	$\gamma_{22} = 8, p_{22}L = 1$

Table 1: An Example to Illustrate the Feedback Allocation

For example, when $F = 3$, γ_{11}, γ_{21} and γ_{12} are selected, so mobile 1 can report 2 link states and mobile 2 can report 1 link state. The feedback allocation with different Fs are shown in Table 2.

	$F = 1$	$F = 2$	$F = 3$	$F = 4$
(m_1, m_2)	(1,0)	(1,1)	(2,1)	(2,2)

Table 2: Feedback Allocation with Different Fs

$\diamond$

Intuition: We know that with the full link state information, the MaxWeight algorithm schedules the set of mobile users to maximize the following weighted sum:

$$\sum_{ij} Q_i(t)X_{ij}(t)Z_{ij}(t). \tag{2}$$

Now with limited feedback, we need to select $Y_{ij}(t)$ carefully so that

$$\sum_{ij} Q_i(t)X_{ij}(t)Y_{ij}(t)Z_{ij}(t)$$

can be close to (2). We therefore prefer those links with large $Q_i(t)X_{ij}(t)$. On average, mobile i has $p_{ik}L$ links with weight $\gamma_{ik} = Q_i(t)r_k$. So we should allow user i to report $p_{ik}L$ link states if $Q_i(t)r_k$ is large enough.

$\diamond$

Next, we analyze the performance of WBF when it is used with the MaxWeight scheduling. The MaxWeight scheduling with limited feedback is as follows:
MaxWeight scheduling: The base station serves mobile i^* over channel j such that

$$i^* \in \arg\max_i Q_i(t)X_{ij}(t)Y_{ij}(t).$$

$\diamond$

We denote by $\vec{Y}^{WBF}(t)$ the feedback allocation decision under WBF at time t and $\vec{Z}^{MW}(t)$ the scheduling decision

under the MaxWeight scheduling at time t. We then define

$$g_p(\vec{Q}(t)) = \mathbf{E}\left[\sum_{i,j} Q_i(t)X_{ij}(t)Y_{ij}^{WBF}(t)Z_{ij}^{MW}(t)|\vec{Q}(t)\right]$$

$$g_f(\vec{Q}(t)) = \mathbf{E}\left[\sum_{i,j} Q_i(t)X_{ij}(t)Z_{ij}^{MW}(t)|\vec{Q}(t)\right].$$

We analyze the performance of WBF+MaxWeight based on the following theorem [7], which reveals the fundamental relation between the efficiency of the proposed algorithm and the value of $g_p(\cdot)/g_f(\cdot)$.

THEOREM 2. *If for some $\epsilon > 0$, the joint WBF feedback and MaxWeight scheduling guarantees*

$$g_p(\vec{Q}(t)) \geq (1 - \epsilon)g_f(\vec{Q}(t))$$

for all $\vec{Q}(t)$, then the joint algorithm can achieve a fraction $(1 - \epsilon)$ of the full network throughput region.

$\diamond$

THEOREM 3. *Given that at most F link states can be reported at each time slot, the joint WBF and MaxWeight scheduling can support a fraction $(1-\epsilon)$ of the full throughput region, where*

$$\epsilon = \frac{(1 - p^{\min})^{\frac{F}{(1+\delta)p^{\max}L}}}{1 - (1 - p^{\min})^{\frac{F}{(1+\delta)p^{\max}L}}} + \tag{3}$$

$$\frac{F}{(1+\delta)(p^{\min})^2 L} \exp\left(-\frac{\delta^2 p^{\min}L}{3}\right). \tag{4}$$

PROOF. To avoid unnecessarily complicated notations, *we assume that at time t, there exists a $\bar{\gamma}(t)$ such that*

$$m_i(t) = \sum_{k:Q_i(t)r_k \geq \bar{\gamma}(t)} (1+\delta)p_{ik}L, \tag{5}$$

i.e., there is no tie in allocating the feedback resource, and mobile i receives exact $(1 + \delta)p_{ik}L$ feedback resource if γ_{ik} is selected. We emphasize that *we make this assumption to simplify notation and our analysis holds without this assumption.* We next introduce a modified WBF, named as MWBF:
Modified Weight Based Feedback (MWBF): Mobile i selects the best $m_i(t)$ links and forms a set named as $\Phi_i(t)$. Link state $X_{ij}(t)$ is reported to the base station if and only if *(i)* $X_{ij}(t) \in \Phi_i(t)$ and *(ii)* $Q_i(t)X_{ij}(t) \geq \bar{\gamma}(t)$.

$\diamond$

We note that the difference between WBF and MWBF is that MWBF will not report link state $X_{ij}(t)$ if $Q_iX_{ij}(t) < \bar{\gamma}(t)$. It is easy to see that if $Y_{ij}^{MWBF} = 1$ then $Y_{ij}^{WBF} = 1$ as well. Therefore, the link states reported under MWBF is a subset of those reported by WBF. Defining

$$g_{mp}(\vec{Q}(t)) = \mathbf{E}\left[\sum_{i,j} Q_i(t)X_{ij}(t)Y_{ij}^{MWBF}(t)Z_{ij}^{MW}(t)|\vec{Q}(t)\right],$$

we have

$$g_{mp}(\vec{Q}(t)) \leq g_p(\vec{Q}(t))$$

holds for all $\vec{Q}(t)$.

We next consider the value of $g_{mp}(\cdot)/g_f(\cdot)$. We first define the event $\mathcal{E}_j$ such that

$$\mathcal{E}_j \text{ occurs if } 0 < \sum_i Q_i(t)X_{ij}(t)Z_{ij}^{MW}(t) < \bar{\gamma}(t).$$

Note that event $\mathcal{E}_j$ occurs when $Q_i(t)X_{ij}(t) < \bar{\gamma}(t)$ for all mobile i. Note MWBF does not report any $X_{ij}(t)$ such that $Q_i(t)X_{ij}(t) < \bar{\gamma}(t)$, so

$$\sum_i Q_i(t)X_{ij}(t)Y_{ij}^{MWBF}(t)Z_{ij}^{MW}(t) = 0$$

when $\mathcal{E}_j$ occurs.

Next we calculate the probability of event $\mathcal{E}_j$. We note that $\mathcal{E}_j$ occurs only if

$$Q_i(t)X_{ij}(t) < \bar{\gamma}(t)$$

for all i. We know that for mobile i, $Q_i(t)X_{ij}(t) < \bar{\gamma}(t)$ occurs when $X_{ij}(t) < \bar{\gamma}(t)/Q_i(t)$, which happens with probability

$$\left(1 - \sum_{k:Q_i(t)r_k \geq \bar{\gamma}(t)} p_{ik}\right).$$

Thus, we have

$$
\begin{aligned}
\Pr(\mathcal{E}_j) &= \prod_i \left(1 - \sum_{k:Q_i(t)r_k \geq \bar{\gamma}(t)} p_{ik}\right) \\
&\leq_{(a)} \prod_i \prod_{k:Q_i(t)r_k \geq \bar{\gamma}(t)} (1 - p_{ik}) \\
&\leq \prod_i \prod_{k:Q_i(t)r_k \geq \bar{\gamma}(t)} \left(1 - p^{\min}\right) \\
&\leq \left(1 - p^{\min}\right)^{\sum_m \sum_k 1_{Q_i(t)r_k > \bar{\gamma}(t)}},
\end{aligned}
$$

where inequality (a) is derived from the fact that

$$\left(1 - \sum_n x_n\right) \leq \prod_n (1 - x_n)$$

when $x_n \geq 0$ for all n.

We note that according to assumption (5)

$$\sum_i m_i(t) = \sum_i \sum_k 1_{Q_i(t)r_k \geq \bar{\gamma}(t)}(1 + \delta)p_{ik}L = F$$

so

$$\sum_i \sum_k 1_{Q_i(t)r_k > \bar{\gamma}(t)} \geq \frac{F}{(1 + \delta)p^{\max}L}.$$

We then conclude that

$$
\mathbf{E}\left[\sum_{i=1}^N Q_i(t)X_{ij}(t)Z_{ij}^{MW}(t) \,\middle|\, \vec{Q}(t), \mathcal{E}_j\right] \Pr(\mathcal{E}_j)
$$
$$
\leq \bar{\gamma}(t)(1 - p^{\min})^{\frac{F}{(1+\delta)p^{\max}L}} \tag{6}
$$

and

$$
\mathbf{E}\left[\sum_{i=1}^N Q_i(t)X_{ij}(t)Y_{ij}^{MWBF}(t)Z_{ij}^{MW}(t) \,\middle|\, \vec{Q}(t), \mathcal{E}_j\right] \Pr(\mathcal{E}_j)
$$
$$
= 0. \tag{7}
$$

We next define event $\mathcal{G}_j$ such that $\mathcal{G}_j$ occurs if both the following two conditions hold:

(i)

$$\sum_i Q_i(t)X_{ij}(t)Z_{ij}^{MW}(t) \geq \bar{\gamma}(t)$$

(ii)

$$
\sum_i Q_i(t)X_{ij}(t)Z_{ij}^{MW}(t)
$$
$$
\neq \sum_i Q_i(t)X_{ij}(t)Y_{ij}^{MWBF}(t)Z_{ij}^{MW}(t).
$$

We note that $\mathcal{G}_j$ occurs only if there exists some link state $X_{ij}(t)$ such that $Q_i(t)X_{ij}(t) \geq \bar{\gamma}(t)$ and $X_{ij}(t)$ is not reported by MWBF, which further implies that there exist a mobile i such that this mobile has more than $\sum_{s=1}^k (1 + \delta)p_{is}L$ links with rate greater than or equal to r_k and $Q_i(t)r_k \geq \bar{\gamma}(t)$ (Mobile i therefore does not have enough feedback resource to report all links with rate greater than or equal to r_k.)

According to Chernoff bound [20], the probability mobile i has more than $m_i(t)$ channels with $X_{ij}(t) \geq \bar{\gamma}(t)/Q_i(t)$ is i,

$$
\begin{aligned}
&\Pr\left(\sum_{j=1}^L 1_{X_{ij}(t) \geq r_k} \geq (1 + \delta)\sum_{s=1}^k p_{is}L\right) \\
&\leq \exp\left(-\frac{\delta^2 \sum_{s=1}^k p_{is}L}{3}\right) \\
&\leq \exp\left(-\frac{\delta^2 p^{\min}L}{3}\right).
\end{aligned}
$$

Note that at most $F/((1 + \delta)p^{\min}L)$ mobile users are allocated non-zero feedback resource because at most $F/((1 + \delta)p^{\min}L)$ different $\gamma_{ik}(t)$ are selected at one time slot, so

$$\Pr(\mathcal{G}_j) \leq \frac{F}{(1 + \delta)p^{\min}L} \exp\left(-\frac{\delta^2 p^{\min}L}{3}\right).$$

Further,

$$\sum_{i=1}^N Q_i(t)X_{ij}(t)Z_{ij}^{MW}(t) \leq (\max_i Q_i(t))r_1$$

holds for all t and j, so

$$
\mathbf{E}\left[\sum_{i=1}^N Q_i(t)X_{ij}(t)Z_{ij}^{MW}(t) \,\middle|\, \mathcal{G}_j\right] \Pr(\mathcal{G}_j)
$$
$$
\leq \left(\max_i Q_i(t)\right)r_1 \frac{F}{(1 + \delta)p^{\min}L} \exp\left(-\frac{\delta^2 p^{\min}L}{3}\right). \tag{8}
$$

Now, note that

$$
\begin{aligned}
&g_f(\vec{Q}(t)) \\
&= \sum_{j=1}^L \mathbf{E}\left[\sum_{i=1}^N Q_i(t)X_{ij}(t)Z_{ij}^{MW}(t) \,\middle|\, \mathcal{G}_j\right] \Pr(\mathcal{G}_j) + \\
&\quad \sum_{j=1}^L \mathbf{E}\left[\sum_{i=1}^N Q_i(t)X_{ij}(t)Z_{ij}^{MW}(t) \,\middle|\, \bar{\mathcal{G}}_j, \mathcal{E}_j\right] \Pr(\mathcal{E}_j, \bar{\mathcal{G}}_j) + \\
&\quad \sum_{j=1}^L \mathbf{E}\left[\sum_{i=1}^N Q_i(t)X_{ij}(t)Z_{ij}^{MW}(t) \,\middle|\, \bar{\mathcal{G}}_j, \bar{\mathcal{E}}_j\right] \Pr(\bar{\mathcal{E}}_j, \bar{\mathcal{G}}_j).
\end{aligned}
$$

We note that $\mathcal{E}_j$ occurs implies that $\bar{\mathcal{G}}_j$ occurs, so

$$
\begin{aligned}
& g_f(\vec{Q}(t)) \\
= & \sum_{j=1}^{L} \mathbf{E}\left[\sum_{i=1}^{N} Q_i(t) X_{ij}(t) Z_{ij}^{MW}(t) \middle| \mathcal{G}_j\right] \Pr(\mathcal{G}_j) + \\
& \sum_{j=1}^{L} \mathbf{E}\left[\sum_{i=1}^{N} Q_i(t) X_{ij}(t) Z_{ij}^{MW}(t) \middle| \mathcal{E}_j\right] \Pr(\mathcal{E}_j) + \\
& \sum_{j=1}^{L} \mathbf{E}\left[\sum_{i=1}^{N} Q_i(t) X_{ij}(t) Z_{ij}^{MW}(t) \middle| \bar{\mathcal{E}}_j, \bar{\mathcal{G}}_j\right] \Pr(\bar{\mathcal{E}}_j, \bar{\mathcal{G}}_j).
\end{aligned}
$$

Further, when $\bar{\mathcal{E}}_j$ and $\bar{\mathcal{G}}_j$ both occur,

$$
\sum_{i=1}^{N} Q_i(t) X_{ij}(t) Z_{ij}^{MW}(t) = \sum_{i=1}^{N} Q_i(t) X_{ij}(t) Y^{MWBF}(t) Z_{ij}^{MW}(t).
$$

To that end, according to (6), (7) and (8), we conclude that

$$
\begin{aligned}
& g_f(\vec{Q}(t)) - g_{mp}(\vec{Q}(t)) \\
\leq & \sum_{j=1}^{L} \mathbf{E}\left[\sum_{i=1}^{N} Q_i(t) X_{ij}(t) Z_{ij}^{MW}(t) \middle| \mathcal{G}_j\right] \Pr(\mathcal{G}_j) + \\
& \sum_{j=1}^{L} \mathbf{E}\left[\sum_{i=1}^{N} Q_i(t) X_{ij}(t) Z_{ij}^{MW}(t) \middle| \mathcal{E}_j\right] \Pr(\mathcal{E}_j) + \\
\leq & L\bar{\gamma}(t)(1 - p^{\min})^{\frac{F}{(1+\delta)p^{\max}L}} + \\
& L\left(\max_i Q_i(t)\right) r_1 \frac{F}{(1+\delta)p^{\min}L} \exp\left(-\frac{\delta^2 p^{\min}L}{3}\right).
\end{aligned}
$$

We also note that the following two inequalities hold

$$
\begin{aligned}
g_f(\vec{Q}(t)) \geq & \sum_{j=1}^{L} \mathbf{E}\left[\sum_{i=1}^{N} Q_i(t) X_{ij}(t) Z_{ij}^{MW}(t) \middle| \bar{\mathcal{E}}_j\right] \Pr(\bar{\mathcal{E}}_j) \\
\geq & L\bar{\gamma}(t)\left(1 - (1 - p^{\min})^{\frac{F}{(1+\delta)p^{\max}L}}\right),
\end{aligned}
$$

and

$$
g_f(\vec{Q}(t)) \geq L\left(\max_i Q_i(t)\right) r_1 p^{\min},
$$

where the second inequality holds because mobile i^* such that $i^* \in \arg\max_i Q_i(t)$ will be scheduled if $X_{i^*j}(t) = r_1$. We then have

$$
\begin{aligned}
& \frac{g_f(\vec{Q}(t)) - g_{mp}(\vec{Q}(t))}{g_f(\vec{Q}(t))} \\
\leq & \frac{(1 - p^{\min})^{\frac{F}{(1+\delta)p^{\max}L}}}{1 - (1 - p^{\min})^{\frac{F}{(1+\delta)p^{\max}L}}} + \\
& \frac{F}{(1+\delta)(p^{\min})^2 L} \exp\left(-\frac{\delta^2 p^{\min}L}{3}\right).
\end{aligned}
$$

Now we define

$$
\begin{aligned}
\epsilon = & \frac{(1 - p^{\min})^{\frac{F}{(1+\delta)p^{\max}L}}}{1 - (1 - p^{\min})^{\frac{F}{(1+\delta)p^{\max}L}}} + \\
& \frac{F}{(1+\delta)(p^{\min})^2 L} \exp\left(-\frac{\delta^2 p^{\min}L}{3}\right).
\end{aligned}
$$

Since $g_{mp}(t) \leq g_p(t)$, the theorem yields from Theorem 2.

$\square$

Remark 1: For the case that $p^{\min} = 0$, Theorem 3 still holds by defining $p^{\min} = \min\{p_{ik} : p_{ik} \neq 0\}$.

Remark 2: To demonstrate the efficiency of WBF, consider the case where $N = L$, which implies that $F \leq L^2$. In this case,

$$
\frac{F}{(1+\delta)(p^{\min})^2 L} \exp\left(-\frac{\delta^2 p^{\min}L}{3}\right)
$$

decreases exponentially in terms of L, and is negligible compared to any constant ϵ when L is sufficiently large. Therefore, for sufficiently large L, to guarantee a fraction $1 - \epsilon$ of the full throughput region, we need to have F feedback resource such that

$$
\epsilon = \frac{(1 - p^{\min})^{\frac{F}{(1+\delta)p^{\max}L}}}{1 - (1 - p^{\min})^{\frac{F}{(1+\delta)p^{\max}L}}}, \tag{9}
$$

which implies that

$$
F = (1+\delta)p^{\max} L \frac{\log \epsilon}{\log(1 - p^{\min})} = \Theta\left(L \log \frac{1}{\epsilon}\right),
$$

which is order optimal because $F = \Theta(L)$ is necessary according to Theorem 1.

Remark 3: In WBF, the base station needs to communicate $m_i(t)$ to mobile i, which incurs an additional communication overhead. This overhead is minor because *(i)* $m_i(t) \leq L$, so only $\log L$ bits are required to represent $m_i(t)$,, and *(ii)* the base station needs to communicates with at most $\frac{F}{(1+\delta)p^{\min}L}$ mobiles at each time slot to convey $\mathbf{m}(t)$. Considering the case $F = \Theta(L)$, the base station only needs to communicates $m_i(t)$ to a constant number of mobiles, and the overall communication overhead of sending $\vec{m}(t)$ is $\Theta(\log L)$, which is a minor communication overhead because the downlink bandwidth is $\Theta(L)$. In fact, this overhead can be further reduced. For example, note that if $\gamma_{ik}(t)$ is "selected" for obtaining the feedback, then $\gamma_{il}(t)$ for $l < k$ is "selected" as well. Therefore, in practical implementations, the base station may just communicate the cut-off link state k_i for mobile i, which requires only $\log R$ bits, and mobile i then can compute $m_i(t)$ locally.

5. SIMULATIONS

In this section, we use simulations to evaluate WBF and compare its performance with a typical feedback allocation scheme.

5.1 Simulation settings

We consider a downlink system with $L = 50$ shared channels and N users. A separate queue is maintained for each user. The packets of flow i arrive at queue i according to a Poisson process with mean arrival rate a_i. Each channel has five possible states (rates) as listed in Table 3.

	r_1	r_2	r_3	r_4	r_5
rate (packets/time slot)	12	9	6	2	1

Table 3: Possible Channel Rates

We evaluate the performance of WBF in two cases: *(i)* link states are i.i.d. across mobile users, named as *i.i.d. case*; and *(ii)* link states are independent across users but not identical, named as *heterogeneous case* (link states for the same user are still assumed to be i.i.d.) .

For the i.i.d. case, we assume the link state distributions are identical and as shown in Table 4.

	r_1	r_2	r_3	r_4	r_5
$\Pr(r_k)$	0.05	0.15	0.15	0.4	0.25

Table 4: Link State Distribution for the I.I.D. Case

For the heterogeneous case, we divide the mobile users into two groups. Users in the same group have the same link state distribution. The distributions are shown in Table 5. We can see that on average the mobiles in Group 2 have better links than those in Group 1. We use Group 2 to represent those mobiles that are close to the base station; and Group 1 to represent those mobiles that are far from the base station.

	r_1	r_2	r_3	r_4	r_5
Group 1 $\Pr(r_k)$	0.05	0.1	0.2	0.4	0.25
Group 2 $\Pr(r_k)$	0.1	0.15	0.4	0.2	0.15

Table 5: Link State Distributions of Group 1 and Group 2

In the following simulations, we compare the proposed WBF with a simple feedback resource allocation scheme which allocates the feedback resource uniformly across users, i.e., each user can report F/N link states. The base station always uses the MaxWeight scheduling based on the reported link states.

5.2 Performance of WBF under Various Traffic Loads

In the first set of simulations, we compare the performance of WBF and the uniform allocation.
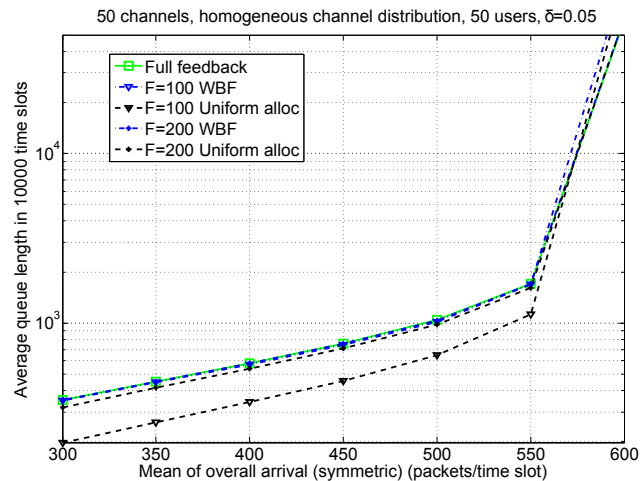

Figure 2: Performance of WBF with Symmetric Arrivals in the I.I.D. Case

We first consider the symmetric case with homogeneous arrivals, which is typical in traditional cellular network when voice call is the dominant service. Figure 2 shows the sum of the queue lengths vs. the sum of arrival rates. Due to the symmetry of both arrival and channel distribution, WBF has a similar performance with the uniform allocation.
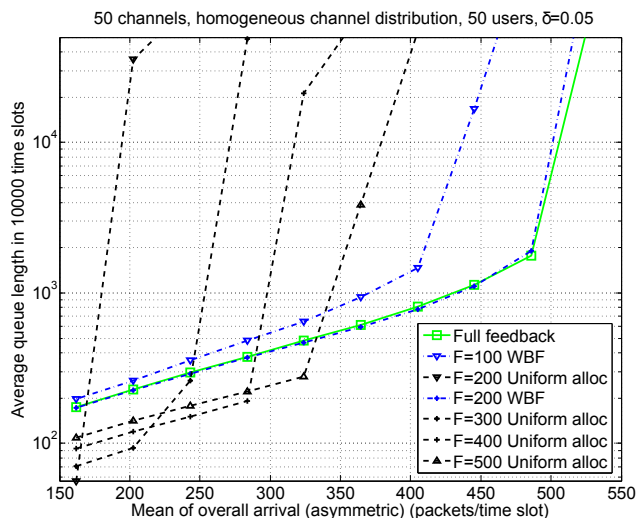

Figure 3: Performance of WBF with Asymmetric Arrivals in the I.I.D. Case

With the emergence of new applications and devices (such as video streaming and smartphones), some users using various new applications might require a higher data rate than others. Hence we consider asymmetric arrivals for the i.i.d. case. We include three classes of users in this simulation: two users with $a_i = 20\alpha$, three users with $a_i = 5\alpha$, and the rest users with $a_i = \alpha$, where α is a positive constant to control the overall traffic load. Figure 3 shows the sum of queue lengths as a function of traffic load. We can see that WBF perform significantly better than the uniform allocation scheme. With $F = 200$, the sum of queue-lengths under WBF is almost identical to that under the full CSI. However, uniform allocation with $F = 500$ is still worse than WBF with $F = 100$ when the traffic load is high. The performance gain comes from the dynamic nature of WBF, which adaptively allocates the feedback resource according to users' demands, which leads to a more efficient resource utilization.

Finally, we consider asymmetric arrivals for the heterogeneous case. Again, we can see in Figure 4 that WBF out-performs the uniform distribution significantly.

5.3 Performance of WBF with Different User Populations

Theorem 3 states that to achieve a fraction $1-\epsilon$ of the full throughput region, WBF needs to acquire at most $\Theta(L \log(1/\epsilon))$ link states at a time, which indicates that the required feedback resource is independent of the user population. We consider case with $30-90$ users and feedback resource $F = 200$ and 300. To verify this result, we explicitly compute the capacity region and choose the arrival rate to be 95% of the throughput limit. Figure 5 shows that as the number of users increases, the network is stable without increasing the feedback resource F, which confirms our theoretical result.

The simulations above validate our analytical results. WBF approaches the full throughput region with a small number of feedbacks, and the required feedback resource is independent of the user population.
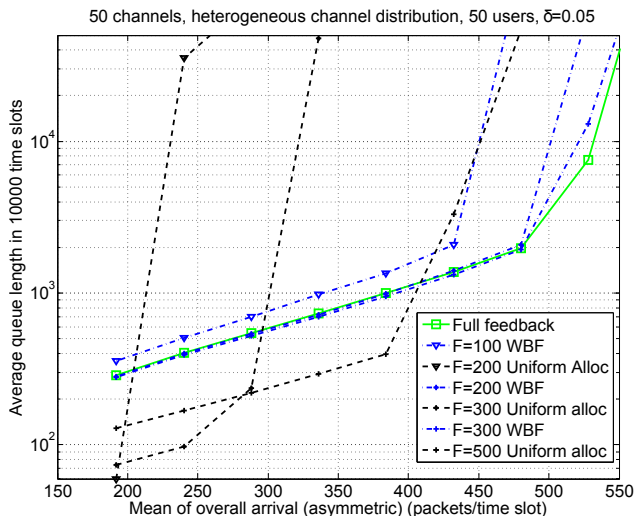
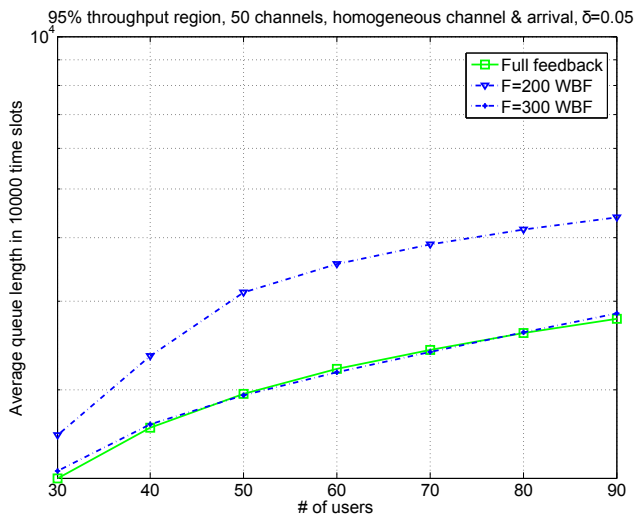**Figure 4: Performance of WBF with Asymmetric Arrivals in the Heterogeneous Case**

Figure 5: Performance of WBF with Different User Population with Symmetric Arrivals in the I.I.D. Case

6. CONCLUSIONS

In this paper, we considered the feedback resource allocation in multichannel wireless downlinks with single base station, L shared channels and N mobile users. We first showed that to support a fraction $(1-\epsilon)$ of the full throughput region (the throughput region with full channel state information), the base station needs to acquire at least $\Theta(L)$ link states at each time slot. We then proposed a feedback allocation algorithm, named WBF. WBF together with the MaxWeight scheduling achieves a fraction $(1 - \epsilon)$ of the full throughput region by acquiring at most $O\left(L \log \frac{1}{\epsilon}\right)$ link states per time slot. Our simulation results demonstrated a significant performance gain of the proposed algorithm compared to the existing approaches.

In this paper, we assumed that a transmission over link (ij) can only be fulfilled if the link state is reported to the base station. Our results however can be easily extended to the case where the base station can communicate with a mobile over unreported links with a base rate r_{base}. In that scenario, we first use WBF to allocate the feedback resource and collect link state information. Then the base station serves mobile j^*, such that

$$j^* \in \arg \max_j (X_{ij}(t)Y_{ij}(t) + r_{base}(1 - Y_{ij}(t)))Q_j(t),$$

over channel i. It can be verified that Theorem 3 still holds given $r_{base} < r_1$. Furthermore, allowing transmissions with a base rate can only improve the throughput of the algorithm, so the order result of Theorem 2 is also valid. Therefore, our order results can be extended to networks where blind transmissions are allowed over unreported channels with a base rate r_{base}.

Another important assumption of this paper is that link states are identically distributed across channels for a given mobile. In practice, it is reasonable to assume that the link states are independent because the channels are orthogonalized, but they may not be identical. One of our future research is to study the case where link state distributions are heterogeneous even for the same mobile user.

Finally, we would like to emphasize that in this paper, we have focused on optimization problem

$$\max \mathbf{E}\left[\sum_i Q_j(t)Y_{ij}X_{ij}\right]$$

for given $Q_j(t)$. It is well-known that by solving this queue-length-based algorithm every time slot, the algorithm can properly allocate the resource to the users and stabilize any traffic load that is within the throughput region. In this paper, we only focus on the stability of the network given a set of flows with persistent arrivals. Another future research problem is to consider networks with delay-sensitive flows and develop feedback allocation algorithms that can provide good delay guarantees.

7. ACKNOWLEDGMENTS

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APPENDIX

Notations

N	number of mobile users
L	number of channels
F	feedback resource
$Q_i(t)$	length of the queue for user i at time t
$X_{ij}(t)$	link rate of link (ij) at time t
$Y_{ij}(t)$	feedback decision of mobile i on channel j
$Z_{ij}(t)$	scheduling decision on link (ij) at time t
$D_i(t)$	$\sum_j X_{ij}(t)Y_{ij}(t)Z_{ij}(t)$
r_k	link rate when the link is in state k
p_{ik}	probability of $X_{ij}(t) = r_k$
$p^{\min}$	lower bound on p_{ik}
$p^{\max}$	upper bound on p_{ik}